Journal of Applied Mechanics and Technical Physics, Vol. 43, No. 3, pp. 362–364, 2002

MAXIMUM CHANNEL PRESSURE OF A NANOSECOND PULSED ANODE DISCHARGE IN A KCl CRYSTAL

V. D. Solovei,[1] V. L. Kolmogorov,[1] and Yu. N. Vershinin[2]

UDC 537.529+531.2

The breakdown channel pressure is estimated for the initial stage of breakdown of a KCl *crystal using experimental data on the fracture of* KCl *crystals upon anode pulsed discharges, the known strength of this crystal, and solutions of the corresponding static boundary-value problem of the anisotropic theory of elasticity.*

Introduction. Knowledge of the breakdown channel pressure for pulsed breakdowns of solid dielectrics is required to understand the physical processes occurring in the channel and to solve applied problems (for example, electroimpulsive fracture of solid dielectrics). Extensive studies have been performed of the regularities of pressure variation in the final stage of the process (stage of expansion of the channel), where the breakdown channel closes the discharge gap and the energy delivered into the channel is determined by electric-circuit parameters [1]. Usually, it is assumed that the maximum breakdown channel pressure is reached in the stage of channel expansion. In this case, in the initial stage of breakdown (stage of propagation of the channel), which lasts from the moment of occurrence of the channel to the moment of closure of the discharge gap by the channel, the channel pressure was ignored because the energy input into the channel in this stage is much less than the energy input in the stage of channel expansion.

At the same time, it is known that the initial stage of an anode discharge is characterized by a hypersonic speed of propagation of the breakdown channel and a considerable difference between thermodynamic plasma parameters in the head of the discharge channel and outside it. From this it follows that a pulsed anode discharge can be treated as a variety of a detonation (according to [2], a detonation is the occurrence and existence of a stationary complex "shock wave–energy-release zone"), namely, as an electron detonation [3]. For detonation processes, in turn, the thermodynamic parameters of materials (including pressure) are maximal in the energy-release zone behind the shock-wave front and depend on shock-wave velocity.

According to quantitative assessments based on the electron detonation model, the pressure in the energy-release zone in the initial stage of breakdown far exceeds the pressure at the final stage of breakdown [3].

The goal of the present work was to estimate the maximum channel pressure for the initial stage of anode discharge using a method that does not consider physical phenomena in the breakdown channel.

Experiment. Figure 1 shows a cylindrical sample of KCl single crystal after its breakdown by a nanosecond pulsed anode discharge. The diameter of the sample was $2b = 5 \cdot 10^{-2}$ m, and its height was $5.4 \cdot 10^{-2}$ m. Sharp electrodes were located above the sample (cathode) and below it (anode). Nanosecond high-voltage pulses with an amplitude $U = 220$ kV, a duration of $3 \cdot 10^{-9}$ sec, and an energy content $W \approx 1$ J were shaped by a "Radan" generator to eliminate or reduce to a minimum the effect of the final stage of the breakdown. The discharge channel formed along the axis of the sample in the $\langle 100 \rangle$ crystallographic direction. The length of a complete breakdown channel was close to the length of an incomplete breakdown channel. In the neighborhood of the channel, open cracks formed in the axial plane of the sample that coincided with the $\{100\}$ crystallographic plane. In the region of reliable recording, we measured the diameter of the breakdown channel on the side of the anode ($2a = 4 \cdot 10^{-5}$ m)

[1]Institute of Theoretical Engineering, Ural Division, Russian Academy of Sciences, Ekaterinburg 620219.
[2]Institute of Electrophysics, Ural Division, Russian Academy of Sciences, Ekaterinburg 620016. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 43, No. 3, pp. 24–27, May–June, 2002. Original article submitted October 24, 2001; revision submitted January 10, 2002.

 0021-8944/02/4303-0362 $27.00

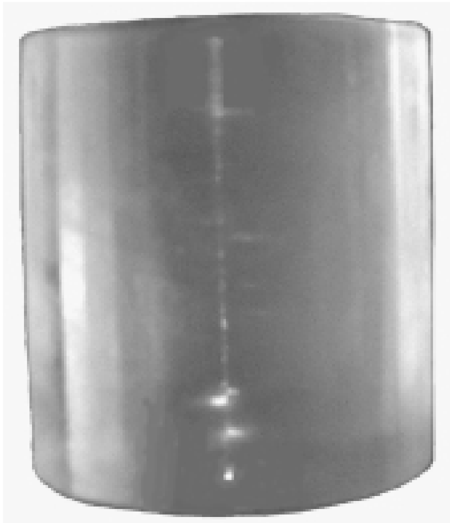

Fig. 1

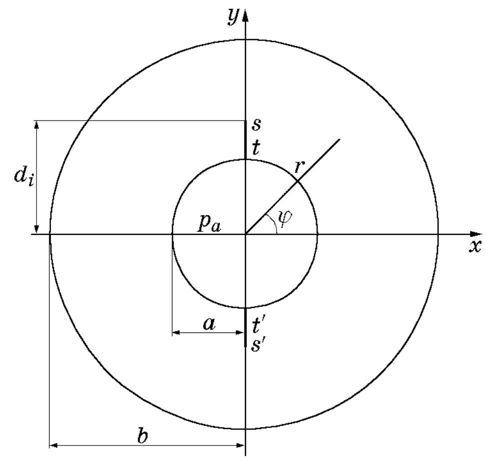

Fig. 2

Fig. 1. Single crystal of KCl after the action of a pulsed anode discharge of voltage 220 kV.

Fig. 2. Diagram of the cross section of the KCl crystal: st and $s't'$ are traces of the cracks and s and s' are the crack tips.

and the distance from the tips of the two longest cracks located near the lower end surface of the sample to the axis of the breakdown channel ($d_1 = 4.44 \cdot 10^{-3}$ m and $d_2 = 2.22 \cdot 10^{-3}$ m). The centers of the cracks are at distances from this surface $5.3 \cdot 10^{-3}$ and $9.0 \cdot 10^{-3}$ m, respectively.

We note that under these conditions, the pressure produced in the energy-release zone acts on the channel walls for fractions of a nanosecond. Therefore, it is possible to assume that during discharge, the dielectric undergoes elastic deformation and brittle fracture [4, 5].

Stress Field in the Crystal Sample. The action of an electric pulse on a dielectric produces elastic strain waves in it, i.e., leads to nonstationary deformation of the dielectric. Nevertheless, in the present work, the deformation of the KCl crystal is considered stationary. Apparently, calculations based on the assumption of stationary deformation underestimate the true pressure in the pulsed breakdown channel. Therefore, the obtained results of approximate calculations give the lower bound of the pressure.

In addition, it is assumed that a plane stress state occurs in the sample cross section (Fig. 2). The Cartesian rectangular coordinate lines (x, y) are directed along the $\langle 100 \rangle$ crystallographic axes of the cubic crystal of KCl. The stress distribution in the crystal is obtained from the solution of the second primal problem of the anisotropic theory of elasticity for a ring loaded over the inner contour by evenly distributed, normal forces. Considering the KCl crystal weakly anisotropic, we use the well-known solution of the problem in [6]. The only component necessary for calculations of the strain tensor $\sigma_{\varphi\varphi}$ from this solution in polar coordinates r and φ (Fig. 2) has the form

$$\sigma_{\varphi\varphi} = p_a k[1 + \bar{r}^{-2} - \lambda(3A\bar{r}^4 + B\bar{r}^2 + C\bar{r}^{-4} + D\bar{r}^{-6})\cos 4\varphi], \tag{1}$$

where $k = c^2/(1 - c^2)$, $A = 5(1 + c^2)/(2\delta)$, $B = -3(1 + 4c^2 + 3c^4)/(2\delta)$, $C = -3c^2(1 + c^2)(1 + 5c^2 + c^4)/(2\delta)$, $D = 5c^4(1 + 4c^2 + c^4)/(2\delta)$, $\delta = 1 + 4c^2 + 10c^4 + 4c^6 + c^8$, $c = a/b$, $\bar{r} = r/b$, $\lambda = (\sqrt{|\alpha_1|} - 1)(\sqrt{|\alpha_2|} - 1)$, $\alpha_1 = (-k_1 + \sqrt{k_1^2 - k_2^2})/k_2$, $\alpha_2 = (-k_1 - \sqrt{k_1^2 - k_2^2})/k_2$, $k_1 = c_{11}^2 - c_{12}^2 - 2c_{12}c_{44}$, $k_2 = 2c_{11}c_{44}$, p_a is the pressure on the inner circle of the ring, b and a are the radii of the outer and inner circles of the ring, respectively, λ is the small parameter of anisotropy of the sample material ($|\lambda| < 1$), for which $k_1 > 0$ and $k_1 > k_2$ (for example, for KCl), and c_{11}, c_{12}, and c_{44} are the elastic moduli for materials with cubic symmetry of elastic properties.

For the tested crystal, $c_{11} = 3.980 \cdot 10^{10}$ Pa, $c_{12} = 0.620 \cdot 10^{10}$ Pa, $c_{44} = 0.625 \cdot 10^{10}$ Pa, and $\lambda = -0.811$ [7].

Estimate of the Pressure in the Electric Pulse Breakdown Channel. The breakdown channel pressure p_a is found from relation (1) if in it we substitute the known stress at any point $\sigma_{\varphi\varphi}$ and the coordinates of this point. Such points s and s' are the tips of two cracks (Fig. 2) whose coordinates are equal to $r_s = d_i$ and $\varphi_s = \pi/2$ ($i = 1, 2$); d_1 and d_2 are measured in experiment.

Let us determine the stress at the crack tips $\sigma^s_{\varphi\varphi}$, assuming that at the indicated points, the fracture criterion for crystals, based on the normal stress law [8], is satisfied:

$$\sigma^s_{\varphi\varphi} = \sigma_{\max}, \tag{2}$$

where $\sigma_{\max}$ is the ultimate normal stress (rupture strength) for the {100} crystallographic plane, which does not depend on the type of stress state.

For uniaxial tension without constraint in the $\langle 100\rangle$ direction, the rupture strength of KCl crystals is obtained in [8]: $\sigma_{\max} = 5$ MPa. This value can be used in (2) if the theoretical tensile strength is higher for the case of constrained deformation than in the absence of it. (It is known [9] that this condition is satisfied for all the crystallographic orientations of NaCl crystals.) In this case, the use of the value $\sigma_{\max} = 5$ MPa in calculations should result in underestimated breakdown channel pressure, which agrees with the lower bound obtained.

From relation (1), using experimental values of a, b, and d_i ($i = 1, 2$) and the above-mentioned values of λ and $\sigma^s_{\varphi\varphi}$, we obtain approximate values of the pressure $p'_a \approx 23.97 \cdot 10^{10}$ Pa and $p''_a \approx 6.114 \cdot 10^{10}$ Pa for $i = 1$ and 2, respectively.

We note that calculations ignoring the elastic anisotropy of the KCl crystal ($\lambda = 0$) yield values of $p'_a \approx 23.89 \cdot 10^{10}$ Pa, and $p''_a \approx 6.112 \cdot 10^{10}$ Pa. It is obvious that the elastic anisotropy of the KCl crystal has little effect on the lower bound of the breakdown channel pressure.

Let us compare the obtained estimates of the breakdown channel pressure for the initial stage of an anode discharge with the electro-detonation wave pressure in the energy-release zone before its decrease by a rarefaction wave. It was found experimentally that a voltage pulse $U = 220$ kV produces an oblique electron-detonation wave in the KCl crystal, which moves at a of velocity of about $2.2 \cdot 10^4$ m/sec (see [3, Fig. 7.8]). According to the well-known relations, this wave velocity corresponds to a pressure of about $3 \cdot 10^{11}$ Pa, which has the same order of magnitude as the estimates given above. Theoretical calculations using the electron-detonation approximation and the approximation of a dense nonideal plasma give similar orders of magnitude for the pressure in the head part of the KCl breakdown channel in the initial stage of breakdown: $p_a \approx 18 \cdot 10^{10}$ and $11 \cdot 10^{10}$ Pa, respectively [10].

Conclusions. From experimental data on the fracture of a KCl single crystal using methods of the anisotropic theory of elasticity, static estimates were obtained for the maximum crystal breakdown channel pressure in the initial stage of the breakdown upon a pulsed anode discharge. The elastic anisotropy of KCl crystals is weak, and its effect on the estimate of the breakdown channel pressure is insignificant. The obtained estimate of the breakdown channel pressure for the tested crystal agrees in order of magnitude with data obtained by different methods, and is their lower bound. For more exact estimate of the breakdown channel pressure, it is necessary to use methods of the nonstationary theory of elasticity.

This work was supported by the Russian Foundation for Fundamental Research (Grant No. 01-01-00581).

REFERENCES

1. B. V. Semkin. A. F. Usov, and V. I. Kurets, *Fundamentals of Electroimpulsive Fracture of Materials* [in Russian], Nauka, St. Petersburg (1995).
2. A. M. Prokhorov, V. I. Konov, I. Ursu, and I. N. Mikhénlesku, *Interaction of Laser Radiation with Metals* [in Russian], Nauka–Izd. Akademii, Moscow–Bucharest (1988).
3. Yu. N. Vershinin, *Electron-Thermal and Detonation Processes during Electric Breakdown of Solid Dielectrics* [in Russian], Izd. Ural. Otd. Ross. Akad. Nauk, Ekaterinburg (2000).
4. A. V. Stepanov, *Fundamentals of the Practical Strength of Crystals* [in Russian], Nauka, Moscow (1974).
5. V. M. Finkel', *Physics of Fracture* [in Russian], Metallurgiya, Moscow (1970).
6. S. G. Lekhnitskii, *Anisotropic Plates*, Gordon and Breach, New York (1968).
7. H. B. Huntington, "The elastic constants of crystals," *Solid State Phys.*, **7**, 213 (1958).
8. E. Schmid and W. Boas, *Plasticity of Crystals, in Particular, Metallic Crystals* [Russian translation], GONTI, Moscow–Leningrad (1938).
9. N. H. Macmillan "The ideal strength of solids," in: R. Latanision and J. R. Pickens (eds.), *Atomistic of Fracture*, Plenum Press, New York (1983), pp. 95–164.
10. Yu. N. Vershinin and D. S. Il'ichev, "Mechanism of electron detonation in solid dielectrics at high discharge rates," *Dokl. Ross. Akad. Nauk*, **374**, No. 2, 187–189 (2000).